Notes on Phase Diagrams

The phase diagram is a theoretical construct which allows one to consider a general equilibrium of two related markets. In macroeconomics, phase diagrams are often used to show the interaction between the financial and goods markets through dynamic changes in real output and interest rates. In microeconomics, phase diagrams can help one understand how equilibrating price changes in one market may be an equilibrating or dis-equilibrating force in a related market. Phase diagrams are an important first step in exploring dynamic economics (i.e., how an equilibrium gets from one place to another over time). It is appropriate to introduce this construct at the beginning of a course in microeconomic theory, because it forces one to review carefully the techniques of Marshallian supply and demand modeling.

The first thing to keep in mind about phase diagrams is that there is really nothing new going on. If one can solve a problem in the ordinary supply and demand setting, with only a modicum of care one can solve the problem using a phase diagram. The extra insight this process gives is well worth the effort.

Consider the following two goods:

\[ x = \text{computer hardware} \]
\[ y = \text{computer software}. \]

These goods sell at prices \( p_x \) and \( p_y \), respectively. In each market there is a Walrasian auctioneer who adjusts prices according to the following tatonnement processes.

**Market x**

\[
\frac{\Delta p_x}{\Delta t} = g \cdot [D_x(p_x) - S_x(p_x)]
\]

where

\[ D_x(p_x) = \text{quantity demanded at price } x \]
\[ S_x(p_x) = \text{quantity supplied at price } x \]
\[ t = \text{time} \]
\[ g > 0 \text{ is the speed of adjustment}. \]

**Market y**

\[
\frac{\Delta p_y}{\Delta t} = h \cdot [D_y(p_y) - S_y(p_y)]
\]

where

\[ D_y(p_y) = \text{quantity demanded at price } y \]
\[ S_y(p_y) = \text{quantity supplied at price } y \]
\[ t = \text{time} \]
\[ h > 0 \text{ is the speed of adjustment}. \]

This adjustment process simply states that when there is an excess demand, the auctioneer increases the price; when there is an excess supply, the auctioneer reduces the price. The parameters \( g \) and \( h \) give the
rates at which price responds to a disequilibrium. Note that the rate of adjustment also depends crucially and the magnitude of the excess demand; that is, for a given speed of adjustment parameter, a bigger the disequilibrium corresponds to a more rapid price adjustment. If the excess demand functions are defined as

\[ E_x(p_x) \equiv D_x(p_x) - S_x(p_x) \]

and

\[ E_y(p_y) \equiv D_y(p_y) - S_y(p_y), \]

the adjustment processes are expressed as follows.

Market \( x \)

\[ \frac{\Delta p_x}{\Delta t} \equiv g \cdot E_x(p_x) \]

Market \( y \)

\[ \frac{\Delta p_y}{\Delta t} \equiv h \cdot E_y(p_y). \]

Graphically,

A similar diagram could be drawn for good \( y \) (software).

It should be clear that the markets for hardware and software are related. We are interested in how much each market reacts to a change in the other good's price. Considering first the market for hardware, what might happen (to the supply and demand for hardware) if the price of software increased?

1. The demand for hardware would probably decrease; as people buy fewer computers they will need to purchase less software.
(2) The supply of hardware might decrease as firms producing both hardware and software (e.g., Microsoft and Apple) shift some of their productive resources from hardware production to software production.

(3) Both (1) and (2) occur.

Case 1: The demand shift dominates, with the result that $p_x$ falls.
Case 2: The supply shift dominates, with the result that \( p_x \) increases.

We will assume that case 1 holds; that is, an increase in the price of software (\( p_y \)) leads to a decrease in the market-clearing price of hardware (\( p_x \)). Using the excess demand function, we get the following graphical depiction.
Now suppose the same situation holds for a series of prices $p_y^0$, $p_y^1$, $p_y^2$, $p_y^3$, where $p_y^3 > p_y^2 > p_y^1 > p_y^0$. This would be shown as

**Definition:** When an increase in the price of good $y$ leads to an increase in the market-clearing price of good $x$, we say that good $x$ is a GROSS SUBSTITUTE for good $y$. When an increase in the price of good $y$ leads to a decrease in the market-clearing price of good $x$, we say that good $x$ is a GROSS COMPLEMENT for good $y$.

Note that we have assumed that hardware (good $x$) is a gross complement for software. Note also that it is possible for good $x$ to be a gross complement for good $y$ while, simultaneously, good $y$ is a gross substitute for good $x$. (The supply shift would dominate in the latter case.)

Since we have an auctioneer adjusting market $x$ toward equilibrium, it makes sense to concentrate on market-clearing prices of good $x$. Recall that an increase in the price of good $y$ is assumed to result in a decrease in the market-clearing price of good $x$. Graphically,
The $E_x = 0$ curve shows market-clearing prices of good $x$ (i.e., where the excess demand curve is zero) as a function of the price of software ($p_y$). Since good $x$ is a gross complement for good $y$, the $E_x = 0$ curve is downward sloping. The “plus” sign to the left of the $E_x = 0$ indicates that when the price of good $x$ is below the equilibrium price, the auctioneer will increase the price of good $x$. Similarly, the “minus” sign to the right of the $E_x = 0$ curve indicates the auctioneer calls a lower price when the price of good $x$ is above the equilibrium price of good $x$.

The same exercise could be carried out for the effects of changes in the price of hardware on the market for software. To expedite matters, let’s assume that software is a gross complement for hardware. Graphically,
Plotting the $E_x = 0$ and $E_y = 0$ curves on the same axes completes the phase diagram. The general equilibrium (i.e., the simultaneous equilibrium of the two markets) occurs where $E_x = 0$ and $E_y = 0$ curves intersect.

The arrows in the four regions indicate the direction of price movement. When the arrows (i.e., vectors) are added by putting them end-to-end we obtain the dynamic behavior of prices. Note that $\rightarrow$ plus $\uparrow$ yields $\nearrow$, $\rightarrow$ plus $\downarrow$ yields $\searrow$, $\leftarrow$ plus $\downarrow$ yields $\swarrow$ and $\leftarrow$ plus $\uparrow$ yields $\nwarrow$. The phase diagram above is stable, because the arrows all point toward the center (i.e., the intersection of the two zero excess demand curves.) Point $E$ is the general equilibrium of the system. It is important to note that if the $E_x = 0$ curve is flatter than the $E_y = 0$ curve, the system is not stable. Graphically,
Here the effect of the price of the related good is stronger than the “own price” effect. When the auctioneer in market \( y \) changes \( p_y \) (in an effort to equilibrate her market), she throws the market for \( x \) way out of equilibrium. The other auctioneer tries to equilibrate his market (i.e., the market for good \( x \)), but the resulting change in \( p_x \) throws the market for good \( y \) out of equilibrium. The process continues and the two price move away from the equilibrium prices. Graphically,

Step (1) The auctioneer in market \( y \) increases \( p_y \) in an effort to equilibrate the market.

Step (2) The increase in \( p_y \) throws market \( x \) out of equilibrium. The auctioneer reduces \( p_x \) in an effort to equilibrate the market.
Step (3) The reduced $p_x$ throws market $y$ even farther out of equilibrium.

Step (4) Go back to step (1).

**Exercises**

1. Show that if good $x$ is a gross complement for good $y$ and good $y$ is a gross substitute for good $x$, then the phase diagram is always stable. Give an intuitive explanation of this result.

2. Show that if good $x$ is a gross substitute for good $y$ and good $y$ is a gross complement for good $x$, then the phase diagram is always stable. Give an intuitive explanation of this result.

3. Using the phase diagram constructed above, carefully explain the dynamic effects of the following events:
   a. Consumer income increases. Assume both hardware and software are normal goods.
   b. The government places a per unit tax on software.
   c. The price of computer chips increases.
   d. Computers become more popular with students—colleges require students to purchase computers.

4. Redo problem 3 assuming that good $x$ is a gross complement for good $y$ and good $y$ is a gross substitute for good $x$. 