1. (a) 

Risk-Averse Person 

\[ V(\pi_B C_B + (1-\pi_B) C_G) \]

\[ V(\pi_B V(C_B) + (1-\pi_B) V(C_G)) \]

\[ \bar{c} = \pi_B C_B + (1-\pi_B) C_G \]

Risk-Loving Person 

\[ V(c) \]

\[ \pi_B V(C_B) + (1-\pi_B) V(C_G) \]

\[ V(\pi_B C_B + (1-\pi_B) C_G) \]

\[ \bar{c} = \pi_B C_B + (1-\pi_B) C_G \]
The top panel shows the Von-Neumann-Morgenstern utility function of a risk-averse person. The utility function, \( V(\cdot) \), is concave for a risk-averse person. This means the utility of the expected outcome is greater than the expected value of utility; that is

\[
V(\Pi_8 C_b + (1-\Pi_8) C_g) > \Pi_8 V(C_b) + (1-\Pi_8) V(C_g).
\]

The diagram shows the certainty equivalent, \( \overline{\bar{c}} \). This is less than \( \bar{c} = \Pi_8 C_b + (1-\Pi_8) C_g \) and has the property that person is indifferent between facing the gamble and receiving \( \bar{c} \) with certainty; that is,

\[
V(\bar{c}) = \Pi_8 V(C_b) + (1-\Pi_8) V(C_g).
\]

For the risk averse person the risk premium, \( \bar{c} - \overline{\bar{c}} \), is positive. This is the amount of consumption the person would be willing to give up to have the gamble taken away. You can think of this as the reservation price of insurance.

The lower panel on page one depicts the situation of a person who
is a risk-lover. The convexity of the utility function, \( V(\cdot) \), means the utility of the expected value is less than the expected value of utility; that is,

\[
V(\pi_\beta C_0 + (1-\pi_\beta) C_6) < \pi_\beta V(C_0) + (1-\pi_\beta) V(C_6).
\]

The diagram also shows that the certainty equivalent, \( \bar{c} \), is greater than the expected value of consumption; that is,

\[
V(\bar{c}) > \pi_\beta V(C_0) + (1-\pi_\beta) V(C_6).
\]

This results in a risk premium of \( E - \bar{c} < 0 \). This negative value means the person would be willing to pay up to \( |E - \bar{c}| \) in order to have the opportunity to face the gamble.

(b) The statement is true, but its proof involves several tacit assumptions. The first assumption is that the person's preferences for the contingent claims can be represented by a Von-Neumann-Morgenstern Utility function.

\( \Box \)
That is, for any two mutually exclusive and exhaustive consumptions \((c_b, c_a)\) paired with their probabilities \((\pi_b, 1-\pi_b)\), the utility function has the form:

\[
U(c_b, c_a; \pi_b, 1-\pi_b) = \pi_b V(c_b) + (1-\pi_b) V(c_a)
\]

where \(V(\cdot)\) is unique up to a positive linear transformation (affine transformation).

(Note: Under the VNM axioms of completeness, reflexivity, transitivity, continuity, monotonicity, reduction of complex lotteries to simple lotteries, and substitution, the utility function must have the VNM form and be unique up to an affine transformation.)

With the VNM utility function the marginal rate of substitution has the form:

\[
MRS = -\frac{\frac{\Delta u}{\Delta c_b}}{\frac{\Delta u}{\Delta c_a}} = \frac{\pi_b}{(1-\pi_b)}
\]

To see this, write the total change in utility as
\[ dU = \Pi_B \frac{\Delta V}{\Delta C_B} dC_B + (1 - \Pi_B) \frac{\Delta V}{\Delta C_G} dC_G. \]

Along an indifference curve, the change in utility is zero.

\[ dU = 0 = \Pi_B \frac{\Delta V}{\Delta C_B} dC_B + (1 - \Pi_B) \frac{\Delta V}{\Delta C_G} dC_G. \]

Obtain the MRS by solving for the slope, \( \frac{dC_G}{dC_B} \).

\[ \text{MRS} = \frac{dC_G}{dC_B} = -\frac{\Pi_B}{1 - \Pi_B} \frac{\Delta V}{\Delta C_G}. \]

Next, let \( A \) be the amount of the accident.

\( K \) be the amount of insurance coverage.

\( W \) be the amount of wealth.

\( \gamma \) be the price per dollar of insurance.

In the good state of nature (i.e., no accident) consumption is

\[ C_G = W - \gamma K. \]
In the bad state of nature consumption (i.e., when the accident occurs) is:

$$ C_B = W - A + \gamma K + K. $$

Thus, the insurance contract allows the person to give up $\gamma K$ in the good state of nature to obtain $(1-\gamma) K$ in the bad state of nature. The insurance contract is represented by a budget line with slope

$$ \frac{\Delta C_B}{\Delta C_G} = \frac{-\gamma K}{(1-\gamma)K} = -\gamma. $$

Graphically,

$$ 45^\circ \ (C_B = C_G) $$
Normally, we would impose two rationing constraints:

1. The person cannot be a net seller of consumption in the bad state of nature. This would mean the person is selling insurance.

2. The person cannot insure for more than 100% of the accident. Full insurance occurs when the person chooses \( k = A \). In this case \( C_B = C_0 \).

These two regions are excluded and are shown as dashed regions of the budget line. The following diagram shows an equilibrium in which partial insurance is chosen.
The next piece of the puzzle is to note that a fair contract (i.e., one with zero expected profit) occurs if and only if \( \gamma = \Pi_B \). To see this, note that the insurance company's expected profit is

\[
\Pi_B \left( \text{Profit in the bad state of nature} \right) + (1-\Pi_B) \left( \text{Profit in the good state of nature} \right)
\]

\[
= \Pi_B (\gamma k - K) + (1-\Pi_B) (\gamma k)
\]

\[
= K \left( \Pi_B (\gamma - 1) + (1-\Pi_B) \right) \gamma
\]

\[
= K \left( \gamma \Pi_B - \Pi_B + \gamma - \gamma \Pi_B \right)
\]

\[
= K (\gamma - \Pi_B).
\]

This will be zero if and only if \( \gamma = \Pi_B \).

The consumer equilibrium condition with a fair insurance contract is

\[
\text{Slope of Budget Line} = -\frac{\Pi_B}{1-\Pi_B} - \frac{\Delta V}{\Delta C_g} \text{MRS}. \\
\text{Slope of } \Pi_B = \frac{-\Pi_B}{1-\Pi_B} - \frac{\Delta V}{\Delta C_g}
\]

Cancelling the term \( \frac{-\Pi_B}{1-\Pi_B} \) yields
an equilibrium condition of

\[ l = \frac{\Delta V}{\Delta C_B} \quad \text{or} \quad \frac{\Delta V}{\Delta C_G} \]

\[ \frac{\Delta V}{\Delta C_B} = \frac{\Delta V}{\Delta C_G} \]

With a risk-averse person (i.e., a person with a concave \( V(c) \)), this condition cannot only occur when \( C_B = C_G \). This is the full insurance case. Utility

![Utility function diagram](image)

Here is the equilibrium with an actuarially fair insurance contract.
And here is the executive summary of the argument.

1. Under the VNM axioms utility is of the form $U(c_o, c_B, \pi_B, 1-\pi_B) = \pi_B V(c_o) + (1-\pi_B)V(c_B)$

2. With VNM utility, $MRS = -\frac{\pi_B \Delta V}{\Delta c_o} / (1-\pi_B) \frac{\Delta V}{\Delta c_B}$

3. With the insurance as specified in the model, the budget line has a slope of $-\frac{1}{1-\gamma}$
(4) In consumer equilibrium:

\[-\frac{\Delta V}{\Delta C_B} = \frac{-\gamma}{1 - \gamma} = (1 - \Pi_B) \frac{\Delta V}{\Delta C_G}\]

(5) With fair insurance (i.e., zero expected profit on the contract), \(\gamma = \Pi_B\), and the equilibrium condition requires \(\frac{\Delta V}{\Delta C_B} = \frac{\Delta V}{\Delta C_G}\).

(6) The only way to have equality of marginal utilities in the good and bad states of nature is for the risk-averse person to be at full insurance \((C_B = C_G)\).
Event: \( p_i \uparrow \), Both goods Normal. \( p_2 = 1 \)

2. (a) Equivalent Variation. \( p'_i > p_i \)

\[ EV = m - m' \]

\[ ES = \frac{1}{11} \]

\[ x_i^*(p_i, p_2, m) = \text{Ordinary demand curve} \]

\[ x_i^u(p_i, p_2, u) = \text{Compensated demand curve} \]

Using the ordinary demand curve to approximate the EV overstates the harm of an increase in \( p_i \) by area ABC.
The **Equivalent Variation** is the amount of income, if taken away at the original price, that would put the consumer on the same indifference curve she would end up on at the higher price.

The equivalent variation and its relationship to the ordinary and compensated demand curves are shown in the above diagram. The consumer begins at point A in the upper diagram and at quantity-price combination \((x_i, p_i)\) in the lower quadrant. When the price increases from \(p_i\) to \(p_i'\), the budget line pivots and the consumer reaches a new equilibrium at point C. \((\text{In the lower quadrant the new equilibrium }(x_i', p_i') \text{ is at point } C)\). The line connecting the two points \((x_i, p_i)\) and \((x_i', p_i')\) gives the ordinary demand curve.

Keeping the price constant at \(p_i\) and reducing income to \(m'\) puts the consumer on the same indifference curve as she ends up on after the price increase. This change in income is the equivalent variation, \(EV = m - m'\).

In the lower quadrant, the compensated
demand curve passes through the points \((x, p, 1)\) and \((x', p', 1)\). The change in the area under this curve (shaded green) is called the equivalent surplus. Using calculus,

\[
ES = \int_{p}^{p'} x_i(p, p', u) \, dp.
\]

The ES is exactly equal to the EV.

If the analyst had mistakenly used the ordinary demand curve to approximate the change in welfare, the measure would include the shaded area plus area \(ABC\). (This could be called area \(P\triangle ABC\) or just \(P\triangle AC\).) Using the ordinary demand curve overstates the welfare change by area \(ABC\).
Compensating Variation

Event: \( p_1 \uparrow \). Both goods normal
\( p_2 = 1, \ p_1' > p_1 \)

\[ CV = m'^2 - m^2 \]

compensating surplus

Using the ordinary demand curve to approximate the CV understates the harm of the price cut by area ABC.
The compensating variation is the amount of income, if given at the new (higher) price that would bring the consumer back up to her original indifference curve.

The compensating variation and its relationship to the ordinary and compensated demand curves are shown in the above diagram. The consumer begins at point A and at quantity-price combination \((x, p)\) in the lower quadrant. When the price increases from \(p\) to \(p'\), the budget line pivots inward and the consumer moves to a new equilibrium at point C. (In the lower quadrant, the new equilibrium \((x', p')\) is shown at point C.) The line connecting these two equilibrium points yields the ordinary demand curve, labeled \(X^*(p, p', m)\). With the price at its new (higher) level, \(p'\), increasing income from \(m\) to \(m'\) will just push the consumer back up to her original indifference curve. This change in income is the compensating variation, \(CV = m' - m\). In the lower quadrant, the
compensated demand curve passes through the points \((x_1, p_1)\) and \((x', p', 1)\). The change in the area under this curve (shaded green, \(\Delta A\)) is called the compensating surplus. Using calculus, the compensating surplus is

\[
\int_{p_1}^{p'} \frac{\partial x}{\partial p} (p_1, p_2, u) \, dp.
\]

The compensating surplus is exactly equal to the compensating variation and, thus, provides an exact measure of the welfare change.

If the analyst had mistakenly used the ordinary demand curve to approximate the welfare change, this measure would exclude the area \(ABC\). Using the ordinary demand curve understates the welfare change by area \(ABC\).
3.

(a) \( p = 120 - 2q \)

(b) \( \epsilon = \frac{\Delta q}{q} \cdot \frac{\Delta p}{p} = \left( \frac{P}{q} \right) \left( \frac{1}{\frac{\Delta p}{\Delta q}} \right) \)

Alternatively, \( \epsilon = \frac{\partial q}{\partial \log p} = \frac{dq}{q} \cdot \frac{1}{dp} \).

(c) \[ \epsilon = \left( \frac{P}{q} \right) \left( \frac{1}{\frac{\Delta p}{\Delta q}} \right) \]

\[ = \left( \frac{120 - 2q}{q} \right) \left( \frac{1}{-2} \right) \]

\[ = q - 60 \]

Solving the demand curve for \( q \) yields \( q = 60 - \frac{1}{2} \cdot \frac{P}{q} \).

\[ \epsilon = \left( \frac{P}{q} \right) \left( \frac{1}{\frac{\Delta p}{\Delta q}} \right) \]

\[ = \left( \frac{P}{60 - \frac{1}{2} \cdot \frac{P}{q}} \right) \left( \frac{1}{-2} \right) \]

\[ = \frac{p}{p - 120} \].
(d) Let\[-4 = \frac{q - 60}{q}\]

and solve for \(q\).

\[-4q = q - 60\]

\[-5q = 60\]

\[\boxed{q = 12}\]

Plug this into the inverse demand function to obtain price.

\[\rho = 120 - 2q\]

\[= 120 - 2(12)\]

\[= 120 - 24 = 96.\]

(e) Let \[-1 = \frac{q - 60}{q}\] and solve for \(q\).

\[-q = q - 60\]

\[2q = 60\]

\[\boxed{q = 30}\]

\[\rho = 120 - 2q = 120 - 2(30)\]

\[= 60.\]
4. (a) Minimize \( wL + rk \)
\( \text{Subject to: } y = L^{.4} k^{.5} \)

The Lagrangian is
\[ L(L, k, \lambda) = wL + rk + \lambda(y - L^{.4} k^{.5}) \]

(b) The first-order (Necessary) conditions for this minimization problem are:
\[ \frac{\partial L}{\partial L} = w - \lambda(t^{.4}) L^{-.6} k^{.5} = 0 \]
\[ \frac{\partial L}{\partial k} = r - \lambda(t^{.5}) L^{.4} k^{.5} = 0 \]
\[ \frac{\partial L}{\partial \lambda} = y - L^{.4} k^{.5} = 0 \]

Using the first two of these yields
\[ \frac{w}{r} = \frac{\lambda(t^{.4}) L^{-.6} k^{.5}}{\lambda(t^{.5}) L^{.4} k^{.5}} \]

or \[ \frac{w}{r} = \left( \frac{.4}{.5} \right) \frac{k}{L} \]

This is the condition that the
iso-cost and iso-quants have the same slope. This condition can be written as

$$K = \left( \frac{w}{14} \right) \left( \frac{r}{15} \right)^{1 - 1} L.$$  

Substitute this expression for $K$ into the constraint.

$$y = L^{1/4} \left[ \left( \frac{w}{14} \right) \left( \frac{r}{15} \right) L \right]^{1/5}$$

$$y = L^{9/4} \left( \frac{w}{14} \right)^{9/5} \left( \frac{r}{15} \right)^{-1/5}$$

Solve for $L$

$$L = y^{1/9} \left( \frac{w}{14} \right)^{-1/5} \left( \frac{r}{15} \right)^{1/9}$$

Substitute this expression for $L$ into the above expression for $K$.

$$K = \left( \frac{w}{14} \right) \left( \frac{r}{15} \right)^{-1} \left( y^{9/9} \left( \frac{w}{14} \right)^{9/5} \left( \frac{r}{15} \right)^{-1/5} \right)^{1/9}$$

$$= y^{9/14} \left( \frac{w}{14} \right)^{9/14} \left( \frac{r}{15} \right)^{-4/14}$$

These choice functions are

$$L^*(w, r, y) = y^{1/9} \left( \frac{w}{14} \right)^{9/14} \left( \frac{r}{15} \right)^{-4/14}$$

$$K^*(w, r, y) = y^{1/9} \left( \frac{w}{14} \right)^{9/14} \left( \frac{r}{15} \right)^{-4/14}$$
(c) The cost function is obtained by plugging $L^*(\cdot)$ and $k^*(\cdot)$ into the definition of cost.

\[
C(y, w, r) = w L^*(w, r, y) + r k^*(w, r, y)
\]

\[
= wy^{\frac{1}{9}} \left( \frac{w}{14} \right)^{-\frac{5}{9}} \left( \frac{w}{15} \right)^{\frac{5}{9}} + ry^{\frac{4}{9}} \left( \frac{r}{14} \right)^{-\frac{4}{9}} \left( \frac{r}{15} \right)^{\frac{4}{9}}
\]

\[
= y^{\frac{1}{9}} \left( w \left( \frac{w}{14} \right)^{-\frac{5}{9}} \left( \frac{w}{15} \right)^{\frac{5}{9}} + r \left( \frac{r}{14} \right)^{-\frac{4}{9}} \left( \frac{r}{15} \right)^{\frac{4}{9}} \right)
\]

Note: \( \bar{AIC} = C(y, w, r) / y \)

\[
MC = \frac{dC(y, w, r)}{dy}
\]
5. (a) The firm seeks to minimize the cost of producing a specified level of output. The constrained optimization problem is

\[ \text{Minimize } wL + rK \]

\[ (L, K) \]

Subject to: \[ y^o = f(L, K). \]
Geometrically, the firm wants to get to the lowest isocost line that still touches the \( y^o \) isoquant.

The tangency condition is

\[ \text{Slope of Iso-cost } = \frac{-w}{r} = \frac{-MP_L}{MP_K} = \text{RTS} = \text{Iso-quant} \]

(b) In this case the firm seeks to maximize the output associated with producing at a specified level of cost. The constrained optimization problem is
Maximize \( f(L,K) \) 
\((L,K)\)

Subject to: \( wL + rK = c_0 \).

Geometrically, the firm wants to get to the highest iso-quant curve that still touches the \( c_0 \) iso-cost line.

The tangency condition is

\[
\frac{\text{Slope of Iso-cost}}{\text{Slope of Iso-quant}} = \frac{-\frac{w}{L}}{-\frac{MP_L}{MP_K}} = \text{RTS}
\]

It is the first problem—minimize cost, subject to a given output—that is relevant to the derivation of the cost function.
The firm derives the cost-output input demand functions by solving problem (a). For labor, it would look like this:

The cost function is obtained by substituting $L^*(w, r, y^*)$ and $K^*(w, r, y^*)$ into the definition of cost.

$$C^*(y^*, w, r) = wL^*(y^*, w, r) + rK^*(y^*, w, r)$$

Dividing this by $y$ gives average total cost

$$\text{ATC}^* = \frac{C^*(y^*, w, r)}{y}, \quad \text{MC}^* = \frac{\partial C^*(y^*, w, r)}{\partial y}$$