Research Statement

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My primary research focus is on inverse problems in imaging with applications to science and industry. More broadly, I apply mathematical techniques to model, simulate, and analyze physical and biological systems. Currently, I have active research projects in de-noising of remote sensing images, in image formation for computational photography, and in fast multipole methods for molecular dynamics simulations. While in industry, I worked on modeling and image analysis for laser RADAR systems, with projects ranging from image formation and reconstruction to phenomenology of laser scattering from aerosols. My Ph.D. thesis work used regularization techniques on inverse problems in imaging with applications to nondestructive evaluation of materials in industrial manufacturing processes.

Current and Future Research

- **Denoising for Remote Sensing:** A fundamental source of noise in coherent imaging systems is speckle, the appearance of bright and dark regions due to constructive and destructive interference of coherent laser light [5]. Accurate restoration of images corrupted by this noise is an open problem in optics [10]. Since speckle occurs via convolution of a complex-valued electromagnetic field with a complex-valued noise term [30], the problem is amenable to a nonlinear inverse problems approach [7]. Regularization has been used in related imaging methods to modest effect, but without treating the nonlinearities or the convolution [16, 26, 9, 2]. Progress in solving this inverse problem will lead naturally to work in correcting remote sensing images which suffer from a related complex-valued convolution noise, namely propagation of laser light through atmospheric turbulence [18, 29]. Preliminary work on this subject is joint with Prof. Scott MacLachlan of Tufts.

- **High Dynamic Range Imaging:** The irradiance map of a natural scene (the amount of light entering the input of a camera) spans up to 8–10 orders of magnitude but typical digital cameras record only two or three orders of magnitude. Recovering the response function of the camera allows one to combine low dynamic range images into a single image with the original dynamic range [6, 20]. Determining the response function of a camera system from multiple photographs of a scene captured with different exposure times is a blind inverse problem wherein the forward operator, which maps a natural scene to a captured photograph, is unknown [3, 32]. The inverse problem can be phrased as a total least squares problem for joint estimation of the camera response and the irradiance map. This is joint work with Prof. Misha Kilmer of Tufts. Preliminary results and analysis will be forthcoming.

- **Fast Multipole Methods:** Joint work with Dr. Bradley Alpert of the National Institute of Standards in Technology (NIST) has developed a fast multipole method for high-speed computation of the electrical potential due to molecules suspended in liquid. We adapted the fast multipole method of Rokhlin and Greengard [1, 4] to update the potential after displacing a single charge but without requiring a complete recomputation. In Monte Carlo Markov chain simulations of molecular dynamics performed at NIST, we expect to realize significant speedup and accuracy improvements.

Ph.D. Thesis: An Inverse Problem arising in Vibro-Thermography

The goal in this work was to nondestructively evaluate the quality of spot welds in sheet metal. The problem was modeled as an inverse problem for the heat equation with an unknown source [8, 13].
• **Description of Physical System:** In a technique called vibrothermography [12], an ultrasonic transducer is attached to one of two sheets of metal, fixed to each other by spot welds, and an impulse is applied. The resulting vibrations cause the sheets to rub together wherever they are not fixed. Heat generated through friction diffuses to the top surface where a time-sequence of images is captured by an infrared camera.

• **Mathematical Model:** To model the heat conduction, the heat equation for \( u(x,y,z,t) \) with boundary conditions for the upper and lower surfaces is used. The inverse problem is to solve for the heat source \( f(x,y,t) \) given measured data \( u(x,y,h,t) = g(x,y,t) \).

• **Time-Slice Approach:** Using an eigenfunction expansion and non-dimensionalization one may approximate the original PDE with

\[
\begin{align*}
ut &= u_{xx} + u_{yy} - 2\alpha u + f \\
(1a) \quad u(x,y,0) &= 0
\end{align*}
\]

where \( \alpha = \beta k/h \) and all quantities are dimensionless. Notice that \( u = u(x,y,t) \) now depends on only two spatial variables. Since the images \( \{g_k(x,y)\} \) are taken at discrete times \( \{t_k\} \), one can consider (1) to be over the time interval \([t_k, t_{k+1}]\) with \( u(x,y,t_k) = g_k(x,y) \) replacing (1b). Duhamel's principle then gives

\[
u(x,y,t_{k+1}) = K_1 f_k + K_0 g_k
\]

where \( f_k = f_k(x,y) \) is the source during the interval, \( K_1 f = k_1 * f \) and \( K_0 f = k_0 * f \) where "*" denotes convolution in two spatial variables and

\[
k_1(x,y) := \int_0^{\Delta t} e^{-2\alpha \tau} e^{-(x^2+y^2)/4\tau} \frac{d\tau}{4\pi \tau} \quad k_0(x,y) := \frac{e^{-2\alpha \Delta t} e^{-(x^2+y^2)/4\Delta t}}{4\pi \Delta t}
\]

with \( \Delta t = t_{k+1} - t_k \) (assumed constant for all \( \{t_k\} \)). Computational simulations using (2) to generate data \( \{g_k\} \) display the same qualitative behavior as experiments performed by General Motors Corp. [17], so the approximations used are in practice acceptable.

• **Inversion:** The form in which (2) is stated lends itself naturally to how the data is given. We represent the infrared video frames as two-dimensional functions at discrete times \( g_k(x,y) = u(x,y,t_k) \in L^2(\mathbb{R}^2) \), for \( k = 0, 1, \ldots, N \). Each image is noisy, as infrared sensors and video recording systems are not noise-free. That is, we do not know \( g_k \), but rather are given \( g_k^\delta(x,y) = g_k(x,y) + n_k^\delta(x,y) \) for \( k = 0, 1, 2, \ldots N \) where \( \|g_k^\delta - g_k\| \leq \delta \) for each \( k \). On each timestep we take

\[
g_k^\delta + 1 = K_0 g_k^\delta + K_1 f_k \quad \text{or} \quad K_1 f_k = g_k^\delta + 1 - K_0 g_k^\delta
\]

which implies that the \( (k+1) \)-th data is generated from the \( k \)-th source and the \( k \)-th data. The inverse problem is to solve (3) for each unknown source \( f_k \) given the data \( g_k \) and \( g_k^\delta \). The problem is ill-posed in the sense of Hadamard [15] since inverting the operator \( K_1 \) on noisy data results in solutions which do not depend continuously on the data. To avoid ill-posedness, we generate approximate solutions by regularizing the operator \( K_1 \) to guarantee well-posed behavior.

We find an approximation to \( f_k \) by Tikhonov regularization [15]

\[
f_k^{\lambda,\delta} = \arg\min_{\phi} \left( \|K_1 \phi + K_0 g_k^\delta - g_k^\delta_{k+1}\|_{L^2}^2 + \lambda \|\phi\|_{L^2}^2 \right)
\]

with unique minimizer \( f_k^{\lambda,\delta} = (\lambda I + K_1^* K_1)^{-1} K_1^* (g_k^\delta_{k+1} - K_0 g_k^\delta) \) leaving us a linear system (with regularizing parameter \( \lambda \) and regularization operator \( R_\lambda \)) which we solve numerically via conjugate gradient. Each time step \( k \) is independent, and could be computed sequentially or in parallel.

An error analysis confirms that this procedure is convergent and numerically stable on the time interval \([0,T]\). We take \( f \) to have time-derivative bounded by \( \psi_t \) at every point in \( \mathbb{R}^2 \). Furthermore we take \( f \) to be
smooth, and more precisely, $f = K^* \psi_2^\tau \forall \tau \in [0, T]$ for some $L^2$ function $\psi_2^\tau$ with $\|\psi_2^\tau\|_T \leq \Psi_2$. The total error is

$$\|f^{\lambda, \delta} - f\|^2_{L^2([\mathbb{R} \times [0, T])} = \frac{T}{4\lambda} \left( \delta(1 + e^{\alpha \Delta t}) + \frac{1}{\alpha^2} (1 - (1 + \alpha \Delta t)e^{-\alpha \Delta t}) \|\psi_1\|_{L^2} + \lambda \Psi_2 \right)^2 + \frac{T}{3} \Delta t^2 \|\psi_1\|_{L^2}^2.$$

Notice that as $\lambda, \delta, \Delta t \to 0$, the total error also goes to zero. That is, approaching the case of no regularization, no noise, and infinitely thin timeslices, the true source is recovered exactly.

**Results:** Simulated data images with noise added and reconstructed sources obtained from the procedure above confirm that the method produces stable and valid results. Two artifacts common in ill-posed reconstruction, Gibbs oscillations and noise blow-up, are absent. In all timesteps, the shape of the source is recovered quite nicely. Numerical experiments indicate that the method is robust to significant noise levels (10%) and a variety of source shapes; one example of robustness to noise is presented in Figure 1.

**Findings:** The original physics-based PDE formulation has been replaced by the much simpler time-slice operator formulation to which the theory of inverse problems can be applied. Theoretical analysis of the Tikhonov-regularized problem shows that the procedure is stable, and numerical results further demonstrate the practicality of the approach even in the presence of significant noise. The quality of our results suggests that a method inferring the source size and shape from our regularized solution will be able to determine weld quality.

**Modeling and Image Analysis for Laser RADAR Systems**

My work at Lockheed Martin Coherent Technologies (LMCT) focused on imaging and modeling in laser RADAR remote sensing systems. Several examples are given below.

**Algorithm development for Fourier transform profilometry:** I was the Algorithms Lead on a structured light 3-D face imaging project for biometric identification [22, 23, 24, 25]. I was responsible for fully automatic algorithms and prototype code for the full image processing stream from data capture to reconstruction of a 3-D surface representation of the subject’s face with an overlaid photograph. The sequence of processing steps is shown with illustrative example data in Figure 2. We used a method of 3D shape measurement called Fourier transform profilometry (FTP) [27, 28], which can reconstruct the 3D profile of a surface up to a maximum slope. I developed a novel method to increase the maximum measurable slope by a factor of three without additional cameras or data-frame captures and which is suitable for a system imaging moving subjects. Details of this method are proprietary and a patent application has been filed.
• **Image reconstruction in coherent laser RADAR imaging:** I developed fast optimization methods to restore digital holography [11] remote sensing imagery degraded by atmospheric turbulence [18]. To correct aberrations caused by atmospheric turbulence in the line-of-sight between the object and receiver one can minimize a cost function related to the visual sharpness of the image [21]. Optimization had been performed using the multidimensional simplex (Nelder-Mead) method which requires many evaluations of the sharpness function. The computational complexity of the sharpness calculation is dominated by 2-D FFTs, which become expensive as the image size becomes large. I developed efficient optimization methods which significantly increased operational frame-rate over existing methods [14].

During the 2007–2008 academic year and summer, I mentored two local high-school interns on related polarization holography work. Their poster won regional and state-wide science fair awards.

• **Modeling nonspherical aerosol backscatter for biological agent standoff detection:** I guided investigations into the phenomenology of laser-particle scattering from terrorist and benign bio-aerosols in order to demonstrate that electromagnetic scattering theory predicts the results observed in several field tests by a LMCT-developed remote sensing system. To this end, I developed a modeling tool based on a standard implementation [19] of the extended boundary condition method [31] for far-field electromagnetic scattering from non-spherical particles. This tool is still in use at LMCT and elsewhere at Lockheed Martin.

• **Simulating multiple scattering of photons in the atmosphere:** I developed, from first principles, a high-fidelity Monte Carlo multiple scattering simulation to study the impact of photon-particle scattering on laser beams propagating through the earth’s atmosphere.

• **Optimization of optical subsystem performance:** As part of a large project developing a multi-function coherent laser RADAR system, I led the modeling and optimization efforts for an optical demultiplexing subsystem design.

**Conclusion**

Throughout my education and employment, I have sought out research opportunities of an interdisciplinary nature, interfacing with and building bridges to scientists from other disciplines. A research environment including mathematicians and scientists from diverse backgrounds stimulates me in my own research and broadens my perspective on the applications of mathematics; I aim to continue working in such environments. I will continue to work on inverse problems and scientific computing at the interface of mathematics and the natural sciences with direct application to problems of interest to science, medicine, and industry.
References


